

AN OPTIMIZATION TECHNIQUES ON THE MANAGERIAL DECISION MAKING

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ABSTRACT

Decision making is essential for organizing a Small Scale Enterprise which faces a large number of problems requiring decisions. The Small Scale Enterprises sector has appeared as a domestic device of expansion in several developing and developed economies of the world. The mathematical model designed to help business manager's plan and make the necessary decision to allocate resources. The present paper discusses the optimization techniques (Linear Programming (LP) and Priority Weighted Goal Programming (PWGP) and Chebyshev Goal Programming (CGP)) to optimize Goal constraints of a small scale enterprise. The results illustrate that the decision makers can select the Priority Weighted Goal Programming model as compare as LP and CGP models.

KEYWORDS: Linear Programming, Goal Programming & SCE

Received: Aug 21, 2018; **Accepted:** Sep 09, 2018; **Published:** Nov 17, 2018; **Paper Id.:** IJMPERDDEC201854

1. INTRODUCTION

The important objective of a Small Scale Enterprise (SCE) is to maximise profits. However, a business firm may have some other objectives such as the maximisation of sales or growth of the firm. The SCE emerged as a vibrant and dynamic sector of the country's economy by virtue of their significant contribution to GDP and industrial production management developments. Several mathematical programming models have been applied to assist the production management planning problems. In this criterion Linear Programming and Goal Programming methods are deity multi criteria decision models. Chambers and Charnes (1961) pioneered the development of a deterministic Linear Programming model in assets and liability. Akpan, N. Petal (2016). Application of Linear Programming for Optimal Use of Raw Materials in Bakery. This work utilized the concept of simplex algorithm; an aspect of Linear Programming to allocate raw materials to competing variables in the bakery for the purpose of profit maximization.

Although the fact that, Bushra Abdul Halim (2015), the decision makers stated, multiple criteria in their managerial problems, the Linear Programming model is unable to combine all the criteria simultaneously. Therefore, the Goal Programming technique has been introduced in order to solve multi-objective problems. Ignizio (1976) proposed a Goal programming model to analyse multiple conflicting objectives while taking into account the constraints and preference of the decision maker.

Standard GP models, Cinzia Colapinto (2015), deal with deterministic Goals that are precisely defined. Variants to standard GP models, includes lexicographic GP (LGP) where the model is optimized according to

DM's prioritized choice, and weighted GP (WGP) where positive and negative deviations from Goals can differ according to the importance of the objectives.

The other GP model, Chebyshev goal programming model, is a specific form of the weighted goal programming model. It is known as CGP, because it uses the underlying Chebyshev (L_∞) means of measuring distance. That is, the maximal deviation from any goal, as opposed to the sum of all deviations, is minimized. For this reason CGP is sometimes termed as min-max GP. CGP is the only major variant that can find optimal solutions for linear models that are not located at extreme points in decision space. Therefore, Chebyshev goal programming has the potential to give the most appropriate solution where a balance between the levels of satisfaction of the goals is needed.

2. REVIEW OF THE LITERATURE

The mathematical techniques in Small Enterprises (SCE) play a very important role in planning and solving decision problems. One of the most commonly used mathematical techniques is Linear Programming (LP), which was first used by Waugh (1951) in feed rations. LP is frequently used in cases when one target functions defined and usually minimizes the costs and maximizes the profit. According to Steven J Miller (2007), Linear Programming is a generalization of Linear algebra use in modelling so many real life problems ranging from scheduling airline routes to ship oil from refineries to cities for the purpose of finding an inexpensive diet capable of meeting daily requirements. Miller argued that the reason for the great versatility of Linear Programming is due to the ease at which constraints can be incorporated into the Linear Programming model. Also used LP, Balogun et al. (2012), in production sectors is the problem of management, that many companies are faced with decision relating to the use of limited resources such as manpower, raw materials, capital etc.

The limitation of the LP mathematical approach is optimized one goal at a time, because of this disadvantage LP is not suitable to use in the SCI planning process having more than one goal must be optimized. To avoid this shortcoming, Jernej (2014) some developed models were upgraded by another mathematical approach called weighted Goal Programming (WGP), where the numerous objective functions (goals) were optimized. One of the specific properties of the WGP techniques is using weights for creating the hierarchical tree of the preferred goals and penalty functions to keep the goals in the tolerant bounds. The advantages of linking LP and WGP are that LP is used to minimize or maximize the goals separately, and WGP reaches all of the Goals from the LP sub-models at the same time. A positive feature of the GP philosophy is its simplicity and ease of use, Aouni B. (2001), which justifies its wide popularity for solving multi criteria decision making models in diverse fields. Many More adopted the Goal Programming models for maximizing the production planning problems; Leung and Chan (2009) propose a GP model for aggregate production planning with resource utilization constraint. Amin Aalaei and Hamid Davoud pour discussed Revised multi-choice goal programming for incorporating dynamic virtual cellular manufacturing into supply chain management, In GP, three of the oldest and still most widely used forms of achievement functions are weighted goal programming (WGP), pre-emptive and Min Max (Chebyshev) structure in which the maximum deviation is minimized. Mohammad Ali Shafia et al. used the integer Chebyshev goal programming for budget allocation in this study uses a type of GP model which has a far better performance in terms of balanced allocation. In this paper, we adopted a Priority Weighted Goal Programming with the percentage normalisation model and a Chebyshev Goal Programming model used to improve the goal values according to the product satisfaction.

3. MODEL FORMULATION

The general **Linear Programming** model [4] with n decision variables and m constraints can be stated in the following form

$$\text{optimize (max or min) } Z = \sum_{j=1}^n c_j x_j \dots (\text{objective function})$$

s.t

$$\sum_{j=1}^n a_{ij} x_j (\leq, =, \geq) b_i, \quad i = 1, 2, 3, \dots, m$$

$$x_j \geq 0, \quad j = 1, 2, 3, \dots, n.$$

Where c_j represent the unit profit (or cost) of decision variables x_j to the value of the objective function.

And a_{ij} represent the amount of resource consumed per unit of the decision variables and b_i represents the total availability of the i^{th} resource. Z represents the measure of performance which can be either profit, or cost or reverence etc.

The LP Problems are single objective oriented problems and the constrained sets of LPP are hard constraints which never accept the violation. Only one single objective is dealt with while in real life situations, problems come with multi-objectives. Under Linear Programming to increase production by a single process the quantity of all inputs is to be increased in a fixed proportion. But the production of a number of goods can be increased to some extent by increasing only one or two inputs. It means that production can be increased to some extent by varying factors proportion. In spite of these limitations, consider the model formulation of Goal Programming.

The general **Goal Programming** model [5] considered as follows.

$$\min z = \sum_{i=1}^m (d_i^+ + d_i^-); \quad i=1, 2, 3, \dots, m \quad (1)$$

st

$$\sum_{j=1}^n c_{i,j} y_j - d_i^+ + d_i^- = A_i; \quad j=1, 2, 3, \dots, n \quad (2)$$

$$\sum_{j=1}^n c_{i,j} y_j \begin{cases} \geq \\ = \\ \leq \end{cases} A_i; \quad i=m+1, \dots, m+p \quad (3)$$

$$(y_j, d_i^+, d_i^- \geq 0) \quad (4)$$

It is important to note that:

Equation (1) refers to the objective function, which is the summation of all deviational variables. Equations (2) and (3) are called Goal and system constraint functions; and they are both referred to Linear constrain function and

Equation (4) is non-negativity constraint.

- m is the number of goals,
- p = number of structural constraints
- n = number of decision variables.
- z = the objective function expressed as the summation of all the deviational variables.
- y_j = the j th decision variables.
- $c_{i,j}$ = the coefficient of the j th decision variables in the i th goal.
- d_i^+, d_i^- are amount of deviation below and above aspiration level respectively.

Also called under achievement and over achievement variables. Therefore, in typical GP model, there are two variables: decision and deviational.

- A_i is the aspiration level

In this paper, it is suggested that **PriorityWeighted Goal Programming** with percentage normalisation is given as GP model. It can be stated mathematically in the following form

$$\min Z = \sum_{i=1}^m \left(\frac{p_i w_i^- d_i^-}{k_i} + \frac{p_i w_i^+ d_i^+}{k_i} \right) w_i^+, w_i^- \geq 0$$

Subject to the constraint functions of equations (2), (3), and non-negativity restriction of (4) and k_i is the normalization factor.

The other model used in this paper is **Chebyshev Goal Programming** has the following algebraic format

$$\min z = \lambda$$

Subject to

$$f_i(x) + d_i^- - d_i^+ = b_i$$

$$\frac{w_i^- d_i^-}{k_i} + \frac{w_i^+ d_i^+}{k_i}, w_i^+, w_i^- \geq 0, i = 1, 2, 3 \dots m.$$

Provided all objective functions $f_i(x)$ are linear, subject to the constraint functions of equations (2), (3), and non-negativity restriction of (4) and k_i is the normalization factor and λ is the maximal deviation from amongst the set of goals.

Therefore, the procedures for achieving a goal are either Minimize the underachievement or Minimize the overachievement or both.

4. METHODS AND MATERIALS

This paper, consider the important optimization techniques of Linear Programming problem and Goal Programming problem. The data for this research project was collected Gorretta bakery limited, Nigeria, Akpan et al. (2016), given in the below table 1. The data consists of the total amount of raw materials (sugar, flour, yeast, salt, and wheat gluten and soybean oil) available for daily production of three different sizes of bread (big loaf, giant loaf and small loaf) and profit contribution per each unit size of bread produced

Table 1: Ingredients and Profit per unit Product Data

Raw Material	Products			
	Big Loaf	Gaint Loaf	Small Loaf	
Profit (N)	30	40	20	20385
Flour (kg)	0.2	0.24	0.14	200
Sugar (g)	0.14	0.2	0.16	160
Yeast (kg)	0.02	0.02	0.02	20
salt(g)	0.0011	0.00105	0.00017	8.5
Wheat Gluten (g)	0.000167	0.002	0.00012	15
soyabean oil (L)	0.015	0.021	0.0098	10

4.1 LP Model Formulation

Let the quantity of big loaf to be produced = x_1

Let the quantity of giant loaf to be produced = x_2

Let the quantity of small loaf to be produced = x_3

Let Z denote the profit to be maximize

The Linear Programming model for the above production data is given by

$$\max Z = 30x_1 + 40x_2 + 20x_3 \leq 20385$$

Subject to

$$0.2x_1 + 0.24x_2 + 0.14x_3 \leq 200$$

$$0.14x_1 + 0.2x_2 + 0.16x_3 \leq 160$$

$$0.02x_1 + 0.02x_2 + 0.02x_3 \leq 20$$

$$0.0011x_1 + 0.00105x_2 + 0.00017x_3 \leq 8.5$$

$$0.000167x_1 + 0.002x_2 + 0.00012x_3 \leq 15$$

$$0.015x_1 + 0.021x_2 + 0.0098x_3 \leq 10$$

$$x_1, x_2, x_3 \geq 0$$

4.2 Result and Analysis of Linear Programming Model

The Linear Programming problem solved by Lingo software. The result is shown in the table 2, the objective function value is 28,385 N, and $x_1=38.0$, $x_2=0$, $x_3=962$ based on this data the optimal solution is derived from the model indicate that two products should be produced 38 units of big loaf and 962 units of small loaves. Their production

quantities should be N 20,385. It seems that the dissatisfaction of a customer chooses at least a giant loaf, for the reason that the customer satisfaction also impacts on regular profits. Overcoming this disadvantage now, consider the Priority Weighted Goal Programming model

Linear Programing Problem by Lingo

Table 2: Solution of LPP

Global optimal solution found.		
Objective value:	20384.62	
Infeasibilities:	0.000000	
Total solver iterations:	3	
Elapsed runtime seconds:	0.46	
Model Class:	LP	
Total variables:	4	
Non Linear variables:	0	
Integer variables:	0	
Total constraints:	7	
Non Linear constraints:	0	
Total non zeros:	21	
Non Linear non zeros:	0	
Variable	Value	Cost
X1	38.46154	0.000000
X2	0.000000	365.0000
X3	961.5385	0.000000

4.3 GP Model Formulation

Here, consider the achievable profit for these products are N 28, 385 and also consider the tolerance limit for these three products should be $x_1=250$, $x_2 = 200$ and $x_3 =550$.

4.3.1 Priority Weighted GP

The priority weighted Goal Programming is modelled as

$$\min Z = P_1 \left(\frac{1}{20385} n_1 \right) + P_2 \left(\frac{1}{250} n_2 + \frac{1}{200} n_2 + \frac{1}{550} n_2 \right)$$

Soft goals are subject to

$$30x_1 + 40x_2 + 20x_3 + n_1 - p_1 = 20385$$

$$x_1 + n_2 - p_2 = 250$$

$$x_2 + n_3 - p_3 = 200$$

$$x_3 + n_4 - p_4 = 550$$

(5)

Hard constrained goals are subject to

$$\begin{aligned}
0.2x_1 + 0.24x_2 + 0.14x_3 &\leq 200 \\
0.14x_1 + 0.2x_2 + 0.16x_3 &\leq 160 \\
0.02x_1 + 0.02x_2 + 0.02x_3 &\leq 20 \\
0.0011x_1 + 0.00105x_2 + 0.00017x_3 &\leq 8.5 \\
0.000167x_1 + 0.002x_2 + 0.00012x_3 &\leq 15 \\
0.015x_1 + 0.0214x_2 + 0.0098x_3 &\leq 10 \\
x_i, n_j, p_j &\geq 0 \quad i=1,2,3. \quad j=1,2,3,4.
\end{aligned} \tag{6}$$

The priority factors; in the Goal Programming algorithm that follows it is assumed that the priority ranking is absolute i.e., P1 goals are more important than P2 goals and P2 goal will not be achieved until P1 goal have been achieved; the same is true for P3, P4 and P5 goals, according to their weights and normalization factor are necessary in order to scale all the goals onto the same units of measurement.

4.3.2 Result and Analysis of Priority Weighted Goal Programming Model

The priority, weighted Goal programming (PWGP) problem solved by Lingo Software. The result is shown in table 3, also $x_1=250.0$, $x_2=4.0$, $x_3=550.0$ based on this data, the Gorretta bakery limited, choose 250 small loafs, 4 giant loafs and 550 big loafs by that optimize their profit, and the objective function value is $z= 1.063958$, it means that the priorities are not achieved, at least one of the goals is not met with an underachievement value $n_2=196$, produce 4 giant loafs and $n_1=1721$, means that the profit is N 18664. But a customer can choose at least a giant loaf.

Priority Weighted Goal Programing Problem by Lingo

Table 3: Solution of PWGP

Global optimal solution found.		
Objective value:	1.063958	
Infeasibilities:	0.000000	
Total solver iterations:	1	
Elapsed runtime seconds:	0.08	
Model Class: LP		
Total variables:	11	
Non Linear variables:	0	
Integer variables:	0	
Total constraints:	11	
Non Linear constraints:	0	
Total non zeros:	36	
Non Linear non zeros:	0	
Variable	Value	Cost
W1	1.000000	0.000000
W2	1.000000	0.000000
W3	1.000000	0.000000
W4	1.000000	0.000000
X(1)	250.0000	0.000000
X(2)	4.095238	0.000000

X(3)	550.0000	0.000000
N(1)	1721.190	0.000000
N(2)	0.000000	0.4974368E-02
N(3)	195.9048	0.000000
N(4)	0.000000	0.2474391E-02
P(1)	0.000000	0.4905568E-04
P(2)	0.000000	0.9743684E-03
P(3)	0.000000	0.5000000E-02
P(4)	0.000000	0.6562096E-03

4.3.3 Chebyshev GP

The Chebyshev goal programming problem modelled as

$$\min z = \lambda$$

Subject to

$$\frac{1}{20385} n_1 \leq \lambda$$

$$\frac{1}{250} n_2 \leq \lambda$$

$$\frac{1}{200} n_3 \leq \lambda$$

$$\frac{1}{550} n_4 \leq \lambda$$

(7)

Also satisfies the soft goals in equations (5) and hard goals (6).

$$x_i, n_j, p_j \geq 0 \quad i=1,2,3. \quad j=1,2,3,4.$$

This model is a specific version of the generic Chebyshev Goal Programming.

4.3.4 Chebyshev Goal Programming Model

The Chebyshev Goal Programming (CGP) problem model solved by Lingo Software. The result is shown in table 4, also $x_1=48.8$, $x_2=39.1$, $x_3=107.5$ based on this data, the Gorretta bakery limited, choose 49 small loafs, 39 giant loafs and 107 big loafs by that optimize their profit, and the objective function value is $z=0.8$, it means that the non of the goals are achieved, $n_1=15203$, means that the profit is N 5182. But the profit of the firm is very less as compare to PW goal programming.

Chebyshev Goal Programing Problem by Lingo

Table 4: Solution of PWGP

Objective value:	0.8044583		
Infeasibilities:	0.000000		
Total solver iterations:	8		
Elapsed runtime seconds:	0.07		
Model Class:	LP		
Total variables:	12		
Nonlinear variables:	0		
Integer variables:	0		
Total constraints:	15		
Nonlinear constraints:	0		
Total nonzeros:	41		
Nonlinear nonzeros:	0		
Variable	Value		Reduced Cost
W1	1.000000		0.000000
W2	1.000000		0.000000
W3	1.000000		0.000000
W4	1.000000		0.000000
LAMBDA	0.8044583		0.000000
X(1)	48.88541	0.000000	
X(2)	39.10833	0.000000	
X(3)	107.5479	0.000000	
N(1)	15203.15	0.000000	
N(2)	201.1146	0.000000	
N(3)	160.8917	0.000000	
N(4)	442.4521	0.000000	
P(1)	0.000000	0.000000	
P(2)	0.000000	0.2933125E-03	
P(3)	0.000000	0.4106375E-02	
P(4)	0.000000	0.1916308E-03	

5. CONCLUSIONS

The analysis presents the optimal solutions for different variations of the LP, PWGP and CGP models. The PWGP and CGP models are providing good enough product satisfactions, but CGP model gives very less profit compare to PWGP model. Also PWGP model is different from the solutions suggested by LP model and CGP model. The LP model suggests a maximum of profit without product satisfaction and CGP model suggests good product satisfaction, but minimum of profit, but the PWGP model suggests the product satisfaction with good enough profit. The PWGP satisfies all the goals; therefore it is more suitable to the Small Scale Enterprises rather than LP and CGP models. In general, the results of this paper make a suitable contribution to understand the possibilities of the Linear Programming, Priority Weighted Goal Programming and Chebyshev Goal Programming as a feasible solution to optimizing processes on a Small Scale Enterprises.

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